The basic cost-volume-profit analysis is widely used for short-term planning, but may not be adequate unless uncertainty is included.

Welsh Hotel: Cost-Volume-Profit Analysis and Uncertainty

Paul A. Phillips

International Journal of Contemporary Hospitality Management, Vol. 6 No. 3, 1994, pp. 31-36 © MCB University Press Limited, 0959-6119

Introduction

Hotels tend to have a high level of fixed costs owing to the levels of investment required. This should result in above normal profits in good times, as variable costs remaining will form a smaller proportion of additional revenue. However, while high profits can be achieved above the break-even point, high losses will result if revenue is significantly reduced.

Thus much attention is given to the traditional CVP model (which ignores uncertainty), as failure to cover fixed costs in the long term can result in the demise of any organization. Is the basic CVP model adequate, bearing in mind that certainty does not always exist during the decision-making process? This article examines the basic CVP model and describes how to include uncertainty during the decision-making process. Prior to any discussion about uncertainty, a review of traditional CVP analysis will be performed.

Traditional CVP Analysis

CVP analysis is an important technique which is widely used for short-term planning purposes. It seeks to examine the relationship between costs, volume, sales, and profit. This makes it a useful managerial tool in a variety of situations, including performing break-even analysis, evaluating pricing strategy, determining special order/booking acceptance or choice of sales mix. The basic equation can be written as:

$$NP = Px - (a + bx)$$

where:

NP = Net profit
P = Selling price per unit
x = Units sold
a = Fixed costs
b = Variable costs per unit.

While the model has several shortcomings owing to its inherent assumptions, which are summarized in Table I, it also assumes that all variable costs are known with certainty. This latter assumption seems peculiar as managers are not making decisions within a vacuum but in an environment that is subject to much turbulence. Thus the overall merits of the model have to be questioned.

The inclusion of uncertainty within the basic CVP model is not a novel idea, as several writers[1-4] have written about the topic. (The term "uncertainty" will be used in a non-mathematical sense as, in practice, the words "risk" and "uncertainty" are often used synonymously.) Jaedicke and Robichek's[2] influential paper specified how the normal probability distribution can be used to show how one variable (price) affects another variable (variable expense), with both being normally distributed, and subject to a known uncertainty in advance. However, there is scant use of probability estimates in decision making. This has been borne out empirically by Drury *et al.*[5, p. 331], who reported, after a study of 300 UK firms, that:

- 49 per cent never used statistical probability analysis for decision making;
- 24 per cent rarely used statistical probability analysis;

Table I. *CVP Assumptions*

- Semi-variable costs can be separated into their fixed and variable elements
- Fixed costs will remain unchanged, and variable costs will vary proportionately with sales volume
- Single product/service, or constant sales mix
- Sales volume is the only factor that affects costs and revenues within the relevant range
- Revenue items vary in direct proportion with volume
- No changes in stock levels or profits are determined on a marginal costing basis
- Efficiency levels remain unchanged

- 20 per cent sometimes used statistical probability analysis;
- 7 per cent often used statistical probability analysis.

While the above study suggests a certain amount of reluctance to use statistical probability estimates, the analysis is not difficult to apply. It is the purpose of this article to show how to apply this methodology to the basic CVP model. Moreover, as most hotels have long-term objectives in terms of return on capital employed (ROCE), emphasis will be placed on sales volume (e.g. rooms sold) and "bottom line" variables. For the purpose of this example, sales price, variable, and fixed costs will be assumed to be certain, while volume is assumed to be uncertain with a normal distribution. This means that the probability distribution for profit can also be assumed to be normal.

The Normal Distribution

The normal distribution is bell-shaped and symmetrical with equal mean and median. To confirm whether a distribution is normal it is usually necessary to ascertain the mean (μ) and the standard deviation (σ). (It ought to be pointed out at this juncture that, even if the precise distribution is not known, the probability can still be determined. Wanhill[6] has shown how the Camp-Meidell inequality can be used to establish the probability of an outcome by using the equation 1/4 × 2. However, the equation suffers if the standard deviation is less than one, and interpolation needs to be performed.) If there is no dispersion, i.e. all observed values are the same, the mean, in this instance, would then be the same as the observed values. Moreover, as dispersions can deviate either side of the mean, it is usually necessary to quantify the amount. To compare two distributions it is necessary to translate the observations of both distributions into Z-values. Basically, Z-values convert each distribution into a standard normal form with a mean of zero, and a standard deviation of one. The formula used being:

$$Z = \frac{X - \mu}{\sigma}$$

where X = Value of variable

μ = Mean value

σ = Standard deviation.

For example, if a variable, X, that has a normal distribution with a μ of 200, and a σ of 20, has an actual observation of 230, the Z-value, when calculated, can be used to establish the probability of this occurrence. Using the above equation, Z can be calculated as follows:

$$Z = \frac{230 - 200}{20} = 1.5$$

The Z-value of 1.5 is shown diagrammatically in Figure 1, and since the total area under the normal distribution curve is equal to one, the shaded portion is equal to the probability of obtaining a value from the original distribution more than 1.5σ from the mean. To convert the Z-value of 1.5 into the actual probability estimate, it is necessary to use normal distribution tables. The probability estimate can be derived from Drury's Appendix C[5] by moving down the left-hand column to "1.5", and moving horizontally to the column headed "0.00". The figure of 0.0668 is equal to the probability of obtaining a value from the original distribution more than 1.5σ from the mean. Conversely, the probability of obtaining a value less than 1.5σ from the mean is 1–0.0668, i.e. 0.9332. Finally, Figure 2 reveals that approximately, 99.7 per cent, 95.4 per cent and 68.3 per cent of total observations lie within 3, 2 and 1 standard deviations respectively.

The Welsh Hotel

Table II shows the 1992 income statement for a hypothetical 125-room hotel in Wales, the data being

Figure 1. *Z-value of 1.5 Shown Diagrammatically*

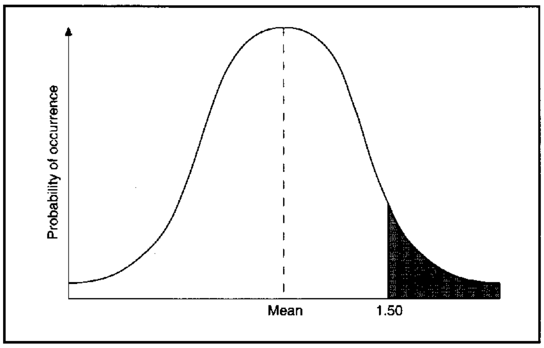


Figure 2. *Observations to Be Found under the Normal Distribution Curve*

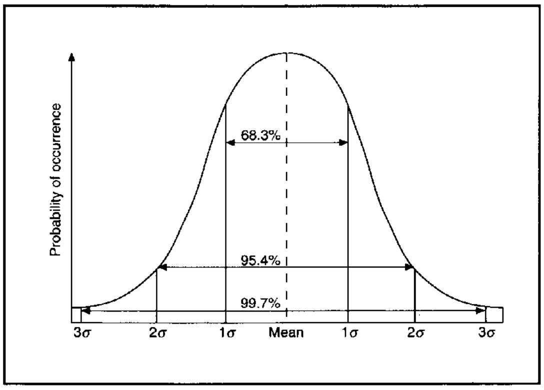


Table II. *1992 Welsh Hotel Income Statement*

Department	Sales (£)	Cost of sales (£)	Payroll and expenses (£)	Total dept expenses (£)	Dept profit (£)
Rooms	881,500		282,125	282,125	599,375
Food and beverage	1,025,375	345,375	351,125	696,500	328,875
Minor operating depts	95,584	24,827	11,507	36,334	59,250
	2,002,459	370,202	644,757	1,014,959	987,500
Less: undistributed operation expenses					
Admin. and general			109,285		
Marketing			55,159		
Energy			77,325		
Property operation			57,736		515,500
Fixed charges			215,995		
Net income before tax					472,000

derived from the Welsh section of the 1992 Horwath Consulting UK hotel report[7], and prepared using the American Uniform System of Accounts for Hotels as a basis. The Welsh hotel's occupancy percentage for 1992 was 57 per cent, which equates to 26,006 rooms being sold for the period. To perform CVP analysis from an income statement it is necessary to prepare a contribution income statement[8, pp. 92-113].

Contribution is equal to sales minus variable costs. In order to determine variable costs it is necessary to separate total costs into their fixed and variable cost components. Obviously, cost of sales, direct wages, and direct expenses are intimately related to sales volume, and are assumed to be variable in nature. However, costs such as managerial salaries and departmental expenses (including energy, telephone, training and marketing costs) will be neither wholly fixed nor variable, and are known as semi-variable costs. Powers[9] stated that, if an expense is to be considered semi-variable, it must pass the following three tests:

(1) It must have a base level of expense. Regardless of sales volume this expense level will occur.

(2) Beyond this base level, the level of expenses will increase with an increase in sales.

(3) The level of increase with changes in sales volume is less than that for variable expense in the same problem situation.

Table III shows the contribution income statement, which for illustrative purposes was prepared after separating semi-variable costs into their fixed and variable elements by apportioning them 80/20 respectively. Naturally, it is up to the individual to obtain the actual split for their operation, by making use of one of the various techniques for determining the proportions of fixed and variable expenses. Such methods include: high-low, least squares, and regression analysis[8, pp. 31-5].

It should be noted that the Welsh hotel is a high fixed cost operation. If we consider sales volume, variable costs amount to 24.9 per cent and fixed costs 51.5 per cent. With regards to total cost, variable cost represents 32.6 per cent, and fixed costs 67.4 per cent. This high percentage of fixed costs results in a high degree of profit instability for the Welsh hotel.

Now that we have separated fixed from variable costs, it is possible to calculate the contribution per room sold:

Using: $\dfrac{\text{Contribution}}{\text{No. of rooms sold}}$

$$= \frac{£1{,}503{,}306}{26{,}006} = £57.81.$$

Once this has been calculated the break-even point can be established. This can be seen graphically in Figure 3, and calculated:

Using: $\dfrac{\text{Fixed costs}}{\text{Contribution per room}}$

$$= \frac{£1{,}031{,}306}{£57.81} = 17{,}840 \text{ rooms sold.}$$

Having previously stated that the number of rooms sold follow a normal distribution, the general manager (GM), in collaboration with his head of department (HOD) for the rooms division has to decide on a figure for the number of rooms sold (the mean), so that there is a 50/50 chance of the actual number of rooms sold being above or below this mean figure. Suppose that, after the preparation of the annual budget, the figure of 26,006 rooms is chosen.

Table III. *1992 Welsh Hotel Contribution Income Statement*

Department	Sales (£)	(%)	Variable cost (£)	(%)	Contribution (£)	(%)
Rooms	881,500	100	56,425	6.4	825,075	93.6
Food and beverage	1,025,375	100	415,600	40.5	609,775	59.5
MOD	95,584	100	27,128	28.4	68,456	71.6
	2,002,459	100	499,153	24.9	1,503,306	75.1
Less:						
Undistributed operation expenses			515,500			
80 per cent of departmental expenses			515,806		1,031,306	51.5
Net income before tax					472,000	23.6

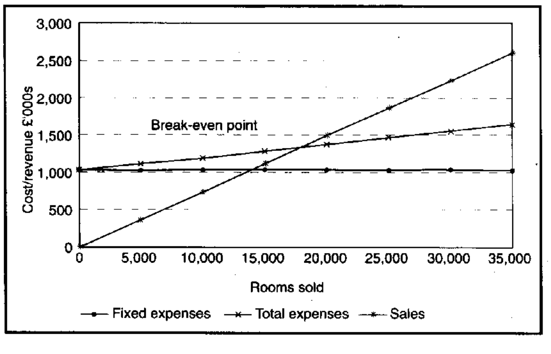
In this example, it happens to be the number of rooms sold the previous year (again, it is up to the reader to determine the appropriate figure). Once the mean has been established the standard deviation can be considered.

To establish the standard deviation in terms of rooms sold it is necessary to apply more probability theory. Based on past experience, the GM and his HOD for rooms decide that there is a 50/50 chance that the final number of rooms sold will vary by 2,000 either side of the mean (see Figure 4). Since approximately 50 per cent of the area under a normally distributed curve lies within +/– 0.67σ from the mean (see Figure 2), then 1σ must be equal to 3,000 rooms. Now that we have satisfied the requirements for a normal distribution, we are in a position to establish the probabilities of different profit levels. Suppose that the GM of the Welsh hotel in collaboration with his assistant manager wants to ascertain the probability of breaking even and making £500,000, £600,000, or £700,000 next year.

(1) *The probability of at least breaking even.*

$$\text{Using: } Z\text{-value} = \frac{17{,}840 - 26{,}006}{3{,}000} = -2.72$$

Figure 3. *Break-even Chart of the Welsh Hotel*



The break-even point therefore lies –2.72σ from the mean of our standard normal distribution. As the distribution is symmetrical, the areas for the negative and positive values are the same. The probability estimate can be derived from Drury's Appendix[5], by moving down the left-hand column to 2.7, and then moving to the column headed 0.02. The figure of 0.00326 represents the probability of not achieving the break-even point. In other words, the probability of "at least breaking even" is (1 – 0.00326) 0.997, i.e. 99.7 per cent.

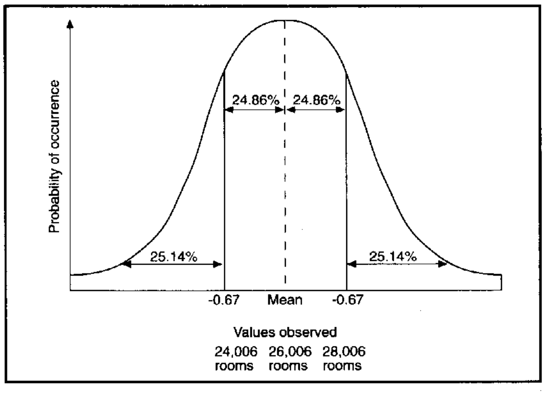
(2) *The probability of at least making £500K.*

The number of rooms that need to be sold to earn a contribution that would produce a net profit of £500K is:

$$\text{Using: } \frac{\text{Fixed costs + Profit requirement}}{\text{Contribution per room}}$$

$$\frac{£1{,}031{,}306 + £500{,}000}{£57.81} = 26{,}489$$

Figure 4. *Normal Distribution for Welsh Hotel Rooms Sold*



$$Z\text{-value} = \frac{26{,}489 - 26{,}006}{3{,}000} = 0.161\sigma,$$

which is equal to a probability of 0.4364, i.e. 43.6 per cent.

(3) *The probability of at least making £600K.*

The number of rooms sold:

$$\frac{£1{,}031{,}306 + £600{,}000}{£57.81} = 28{,}218$$

$$Z\text{-value} = \frac{28{,}218 - 26{,}006}{3{,}000} = 0.737\sigma,$$

which is a probability of 0.2296, i.e. 23 per cent.

(4) *The probability of at least making £700K.*

The number of rooms sold being:

$$\frac{£1{,}031{,}306 + £700{,}000}{£57.81} = 29{,}948$$

$$Z\text{-value} = \frac{29{,}948 - 26{,}006}{3{,}000} = 1.314\sigma,$$

which is equal to a probability of 0.0951, i.e. 9.51 per cent.

From the above results for the Welsh Hotel, it might be prudent for the GM to forget about making £600,000 or more; the hotel has never previously achieved a profit with an associated probability of less than 30 per cent.

This article has shown the hotelier how to incorporate uncertainty into the basic CVP model. The author is of the opinion that any risk-averse GM would benefit from using probability theory during short-term decision making. However, the user should not treat the model as a panacea, as the results themselves will be as good only as the data and assumptions used.

It seems therefore necessary to enumerate some of the inherent operational difficulties of the basic CVP model:

(1) *Cost structure:* Since separating semi-variable costs into their fixed and variable elements is at the heart of CVP analysis, all decision makers ought to be fully aware of, and understand, the cost structure of their operation; otherwise CVP analysis will provide meaningless information.

(2) *Cost behaviour:* The basic model presumes that fixed costs remain fixed and that variable costs per unit remain constant. Nevertheless, costs do not always behave in the usually assumed manner. Fixed costs should not automatically be drawn as a horizontal line, as in reality they may be more "step-shaped", with each ledge of the step representing the range of activity where fixed costs remain constant. This range is also known as the relevant range. In addition, it must be borne in mind that variable costs might be more curvilinear, than linear.

(3) *Sales mix:* Hotels like most other businesses suffer from seasonality, and the profit/volume (P/V) ratio would fluctuate from one sales mix to another. Hence the more varied the sales mix, the greater the problem for the manager.

(4) *Multi-product:* Perhaps one of the most critical issues when considering the operational difficulties of the basic CVP model, is the fact that there is the assumption that a single product/service is sold. In the case of the Welsh hotel, which has more than one revenue-generating department, the user might need critically to assess the contribution of each unit of the hotel. In this instance the user could perform break-even analysis making use of a P/V graph, as illustrated in Figure 5, on a departmental basis.

However, while the Welsh hotel consists of double and twin rooms, which are similarly priced for double or single occupancy. For the hotel which consists of suites, double and single rooms, the hotelier may prefer to perform more detailed room analysis. For illustrative purposes, Table IV shows the potential daily and annual room revenue for Hotel X. The rack rate for the suite (£140), double (£105), and single (£70) rooms, are in the proportions of 2, 1.5 and 1 respectively. Thus, if the rooms sold each night were converted into equivalent units (Eq. Units) of £70, with weightings of 2, 1.5, and 1, for the suite, double, and single rooms, the daily and year-to-date room revenue can be compared with the daily and year-to-date break-even points.

It is also necessary to ascertain fixed costs (e.g., building depreciation, heating, and air-conditioning), and variable costs (e.g. linen and cleaning) for each type of room sold within Hotel X. Let us assume that fixed and variable costs totalled £2 million for the rooms department. As total cost equals total revenue at the break-even point, £2 million of rooms revenue has to be generated

Figure 5. *Profit-Volume Chart of the Welsh Hotel*

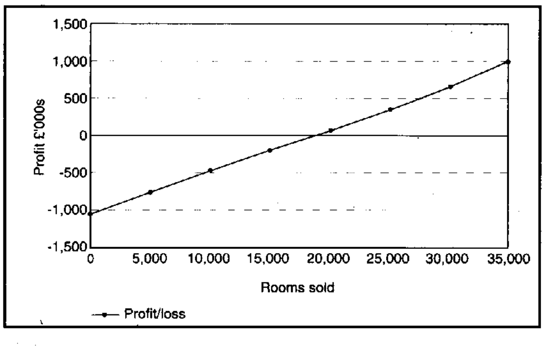


Table IV. *Potential Annual Room Revenue for Hotel X*

Type of room	No. of rooms	Rack rate (£)	Potential daily revenue (£)	Potential annual revenue (£)
Suite	5	140	700	255,500
Double	80	105	8,400	3,066,000
Single	40	70	2,800	1,022,000
	125		11,900	4,343,500

annually for the department to break even. In other words, the break-even occupancy percentage is £2/£4.3 million, i.e. 46 per cent. Thus, if we refer back to Table IV, the daily room revenue needs to be at least 46 per cent of £11,900, i.e. £5,474, or approximately 79 Eq. Units.

The HOD for rooms now knows that, if the hotel sells rooms at the rack rate, it needs to sell at least 79 Eq. Units to break even. It can therefore sell in any permutation, as long as it sells a minimum of 79 Eq. Units. In other words, it could sell 53 double rooms (53*1.5) = 79.5 Eq. Units, or one suite, 25 double rooms, and 40 single rooms (1*2)+(25*1.5)+(40*1) = 79.5 Eq. Units. A similar methodology can also be used for the food and beverage, and minor operating departments.

As noted earlier, the user should seek advice as to the suitability of the CVP model application prior to making important operational decisions, as problems will present themselves differently for each class of hotel.

Conclusion

This article has concentrated on assessing the probability of achieving certain profit levels for the Welsh hotel. However, the methodology could also be applied, assuming normal distribution remains valid, to a range of business and operational issues in any hospitality organization. For example, various services and departments could be assessed to determine the optimum solution for achieving a certain minimum profit.

While the inclusion of uncertainty with the basic CVP model is not contemporary, it would appear that its diffusion rate is at best modest. It is hoped, therefore, that the approach outlined in this article will be tested in other areas of the hospitality industry, especially where the term "uncertainty" can be interpreted and quantified.

References

1. Bierman, H. Jr, *Topics in Cost Accounting and Decisions*, McGraw-Hill, New York, NY, 1963, pp. 36-40, 40-46.
2. Jaedicke, R.K. and Robichek, A.A., "Cost-Volume-Profit Analysis under Conditions of Uncertainty", *The Accounting Review*, October 1964, pp. 917-26.
3. Johnson, G.L. and Simik, S.S., "Multiproduct C-V-P Analysis under Uncertainty", *Journal of Accounting Research*, Vol. 9 No. 2, Autumn 1971, pp. 278-86.
4. Dickinson, J.P., "Cost-Volume-Profit Analysis under Uncertainty", *Journal of Accounting Research*, Vol. 12 No. 1, Spring 1974, pp. 182-7.
5. Drury, C., Braund, S. and Tayles, M., "A Survey of Management Accounting Practices in UK Manufacturing Companies", ACCA Research Occasional Paper, Chartered Association of Certified Accountants, UK.
6. Wanhill, S.R.C., "Which Investment Incentives for Tourism?", *Tourism Management*, Vol. 7 No. 1, March 1986, pp. 2-7.
7. Horwath Consulting, *United Kingdom Hotel Industry 1992*, pp. 38-9.
8. Kotas, R., *Management Accounting for Hotels and Restaurants*, 2nd edition, Chapman & Hall, 1986, pp. 92-113.
9. Powers, T.L., "Break-even Analysis with Semi-fixed Costs", *Industrial Marketing Management*, Vol. 16, 1987, pp. 35-41.

Paul A. Phillips is Lecturer in Financial Management at the School of Consumer Studies, Tourism and Hospitality Management in the University of Wales College of Cardiff, Cardiff, UK.